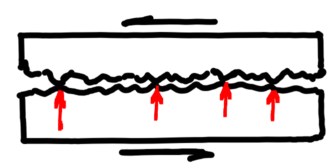
Chapter 7- Surface Failure

The previous chapters have dealt with failure of parts by distortion (yielding), instability (buckling), and breakage (fracture). In addition to these, there are various kinds of damage that can occur to the surface of the part that can render it unfit for use. In fact, experience indicates that more machine parts fail through surface damage than by breakage. In an automobile, for example, exhaust systems corrode, body panels rust, piston rings, suspension joints, and other contacting parts "wear" out. Wear is a broad term that encompasses many types of failure due to surface damage. It is often divided into several categories such as adhesive wear, abrasive wear, corrosion wear, and surface fatigue. The latter is discussed in Sections 7.7-13. The other categories are covered in Sections 7.2-6.

Adhesive wear- On a microscopic scale, sliding metal surfaces are never smooth. Although surface roughness may be only a few hundredth of a micrometer, peaks (called "asperities") and valleys occur, as shown. Since the force of contact and frictional heat of sliding are concentrated at a small area of contact, welding takes place at these contact points. To permit relative motion, either these welds or one of the two metals near the weld must fail in shear. New welds (adhesion) and fractures continue to occur, resulting in adhesive wear. If local welding of asperities is so extensive that sliding stops, the resulting failure is called seizure. If welding/tearing causes a transfer of metal between surfaces, the resulting wear is called scoring. Severe adhesive wear is also called galling. Mild adhesive wear between piston rings & cylinder walls is called scuffing.



Greatly enlarged view of two "smooth" rubbing surfaces

Abrasive wear involves a hard, rough surface abrading material from a softer one, or loose, hard particles trapped between two surfaces and abrading both.

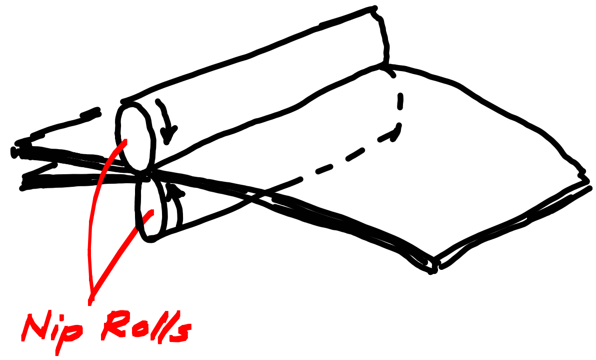
Corrosion wear occurs when a corrosive atmosphere (such as oxygen) is present to oxidize the surface of the material. Sliding breaks the oxides and other contaminants free from the surfaces. This exposes new material to the corrosive elements and also turns the often hard corrosion products into abradants. The combination of a corrosive environment with cyclic stresses greatly shortens the fatigue life of materials and leads in corrosive fatigue failure. In tight joints (such as press fits) where no gross motion is present, tiny vibratory motions are sufficient to set up a corrosive wear process called fretting that can remove significant volumes of material over time.

Reading Assignment: Sections 7-0 to 7-6.

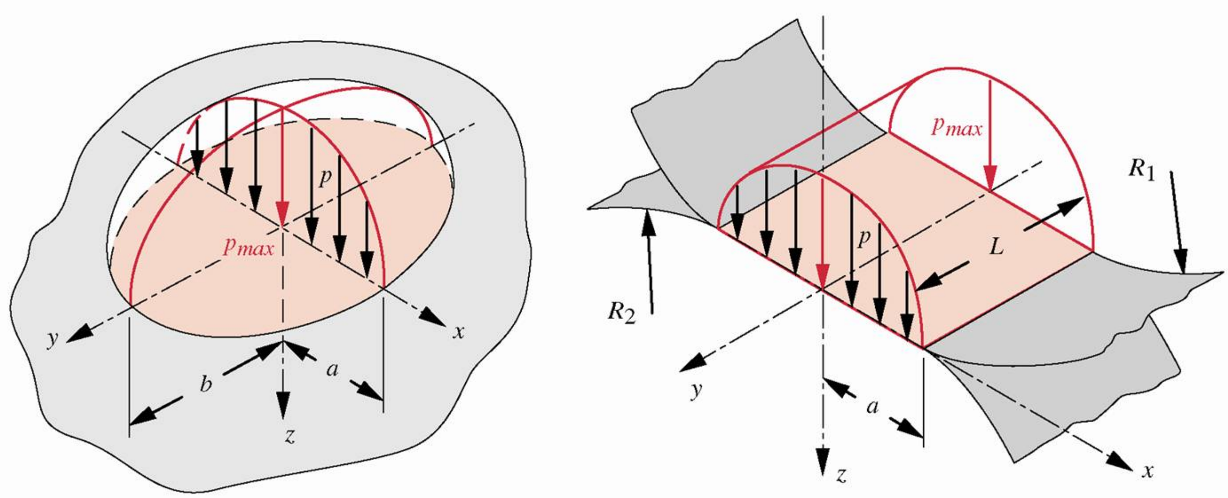
7.7 Surface fatigue occurs in pure rolling or rolling-sliding contact, but not in pure sliding situations as was the case in various types of wear we discussed above. Ball and roller bearings, cams and roller followers, and nip rolls, for example, are subject to pure rolling while spur and helical gear tooth contact are subject to rolling and significant sliding at portions of their tooth interface. As a result all of these are subject to surface fatigue.

What are Nip Rolls?

Nip rolls are powered rolls that are used to press two or more sheets together to form a laminated product. The high pressure created at the nip point brings the sheets into contact, and can squeeze out any bubbles or blisters that might cause a defective bond. Nip rolls can be used to laminate sheets using adhesives, or contact cement. Nip rolls are sometimes called laminating rolls, laminators, squeeze rolls, or pinch rolls.



The theoretical contact area between the two cylinders, or between the cylinder and a flat surface is just a line of zero width (similarly the contact area between two spheres is zero). So the applied forces create an infinite stress. We know that this cannot be true, as the material deforms to create sufficient contact area to support the load at some finite stress. In the general case, the distribution of this stress over the contact area is semi- ellipsoidal and the contact area is elliptical (Fig. 7-11a). Spheres will create a circular contact area, and cylinders create a rectangular contact area (Fig. 7-11b).

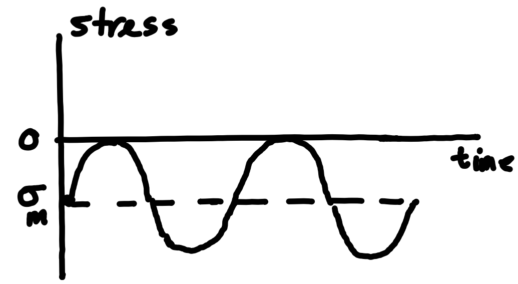


(a) Ellipsoidal pressure distribution in general contact—for spherical contact $a = b$

(b) Ellipsoidal-prism pressure distribution in cylindrical contact

Figure 7-11
Pressure Distributions and Contact Zones of Spherical, Cylindrical, and General Hertzian Contact.

Consider a spherical ball rolling without slipping on a surface under a constant normal load. If the stress produced by the load is below the material's yield point, the deformation in the contact area will be elastic and the surface will return to its original geometry after ball passes through the point. The resulting stresses are called contact stresses or Hertzian stresses. These contact stresses are repeated at the ball's rotation frequency. This creates a fatigue loading that eventually leads to a surface fatigue failure.



Principal contact stresses give rise to shear stresses which cause cracks after many stress-cycles. Crack growth then eventually results in failure by pitting-the fracture and dislodgment of small pieces of material from surface. Once the surface begins to pit, its surface finish is compromised and it rapidly proceeds to failure by spalling - the loss of large pieces of the surface.

If the load is large enough to produce stresses higher than the materials compressive yield strength, then the contact area will permanently deform into a flat area. This is referred to as false brinelling because of its similarity to the indentation made in a Brinell hardness test. Such a flat surface on even one ball (or roller) makes a ball (or roller) bearing useless.

7.8 Spherical Contacts

We investigate the contact area geometry, stress distribution, and deformation in rolling contact of sphere-on sphere. The equations were mainly derived by Hertz in 1881. Cross-sections of two spheres in contact are shown in Fig. 7-13a.

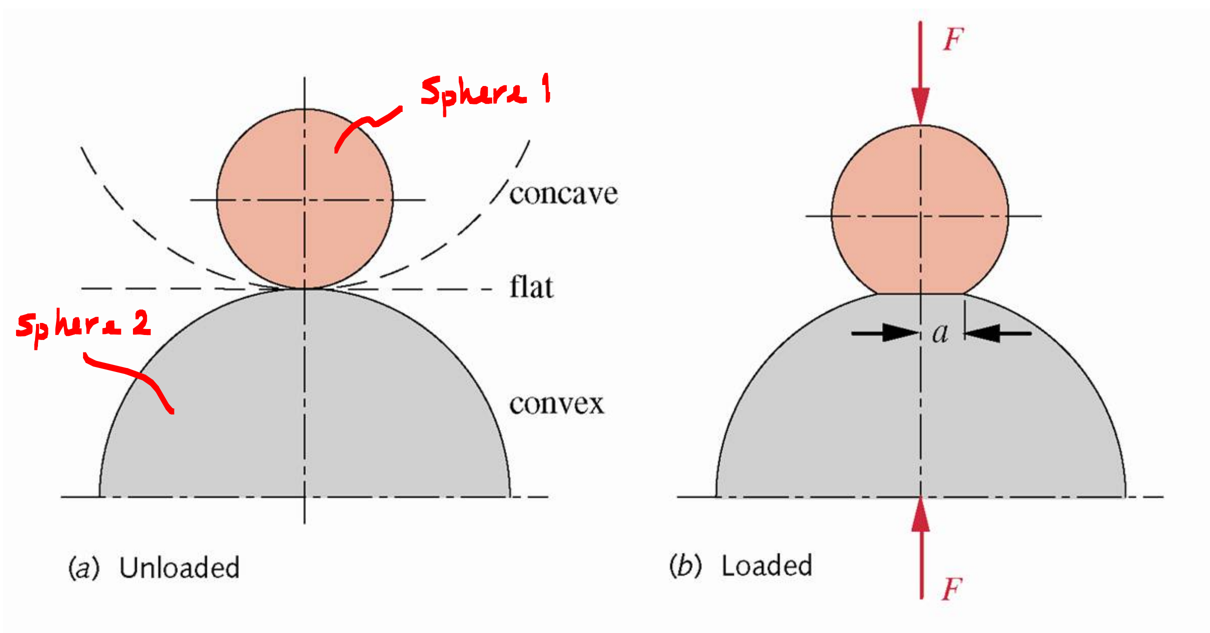


Figure 7-13

The dotted lines show the possibilities that one is a flat plane or a concave surface. The difference is the sign of radius of curvature (convex > 0, concave <0, flat = 0).
After loading the point contact changes to a circular contact area. Hertz has shown that the radius "a" of the contact area is:

$$a=\sqrt[3]{\frac{3F}{8}\;\frac{(1-\nu_1^2)/E_1+(1-\nu_2^2)/E_2}{1/2R_1+1/2R_2}}$$

where E_1, E_2, ν_1, ν_2 are the Young's moduli and Poisson's ratio, for the materials of sphere 1 and sphere 2; and where R_1 and R_2 are the radii of the spheres. Defining

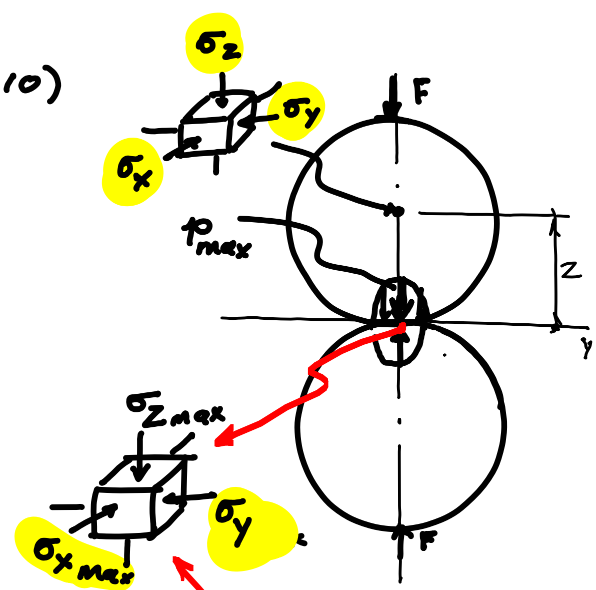
$$m_1=\frac{1-\nu_1^2}{E_1}\quad,\quad m_2=\frac{1-\nu_2^2}{E_2}\quad,\quad B=\frac{1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\qquad(7.9a,b)$$

the equation for the radius of contact area becomes

$$a=\sqrt[3]{0.375\,\frac{m_1+m_2}{B}\,F}\qquad(7.9d)$$

The contact pressure distribution is ellipsoidal. It is a maximum at the center and zero at the edge:

$$p=p_{max}\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{a^2}}\qquad(7.10)$$



where

$$p_{max}=\frac{3}{2}\frac{F}{\pi a^2}\qquad(7.8b)$$

The average pressure is

$$p_{ave}=\frac{F}{area}=\frac{F}{\pi a^2}$$

$$\therefore\quad p_{max}=\frac{3}{2}p_{ave}\qquad(7.8d)$$

The applied force F creates three compressive stresses σ_x, σ_y, and σ_z.
Using the concepts of the theory of elasticity (see Reference 19 of text),
we can solve for these stresses. If we look at these stresses as they vary along the z-axis (with z increasing into the material), we find

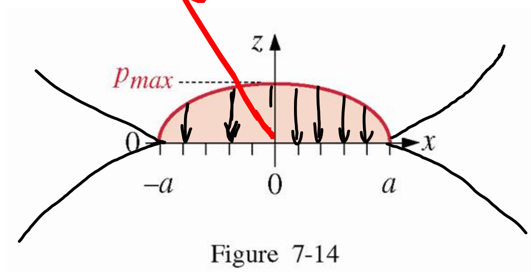


Figure 7-14
Pressure Distribution Across Contact Patch.

$$\sigma_z = p_{max}\left(\tilde{Z}^3 - 1\right) \qquad (7.11a)$$

$$\sigma_x = \sigma_y = \frac{p_{max}}{2}\left[-(1+2\nu) + 2(1+\nu)\tilde{Z} - \tilde{Z}^3\right] \qquad (7.11b)$$

where $\tilde{Z} = \dfrac{z/a}{\sqrt{1+(z/a)^2}}$, and the Poisson's ratio is taken for the sphere of interest.

The maximum shear stress is obtained from the principal stresses (7.11 a) and (7.11b):

$$\tau_{13} = \frac{1}{2}\left(\sigma_x - \sigma_z\right) = \frac{p_{max}}{2}\left[\frac{1-2\nu}{2} + (1+\nu)\tilde{Z} - \frac{3}{2}\tilde{Z}^3\right] \qquad (7.12a)$$

The principal stresses σ_x, σ_y, and σ_z given by (7.11) are maximal on the surface of spheres at z = 0 while τ_{13} is a maximum at a small distance below the surface (denoted by z at τ_{max}). These maximal values are obtained from (7. 11) and (7-12) as:

$$\sigma_{z\,max} = -p_{max} \quad , \quad \sigma_{x\,max} = \sigma_{y\,max} = -\frac{1+2\nu}{2}p_{max} \qquad (7.11c,d)$$

and

$$\tau_{13\,max} = \frac{p_{max}}{2}\left[\frac{1-2\nu}{2} + \frac{2}{9}(1+\nu)\sqrt{2(1+\nu)}\right] \qquad (7\text{-}12b)$$

which occurs at

$$z@\tau_{max} = a\sqrt{\frac{2(1+\nu)}{7-2\nu}} \qquad (7.12c)$$

A plot of the principal stresses σ_x, σ_y, σ_z , and the maximum shear stress τ_{13} as a function of normalized depth z/a is shown in Fig. 7-15 (both materials are steel, ν = 0.28).

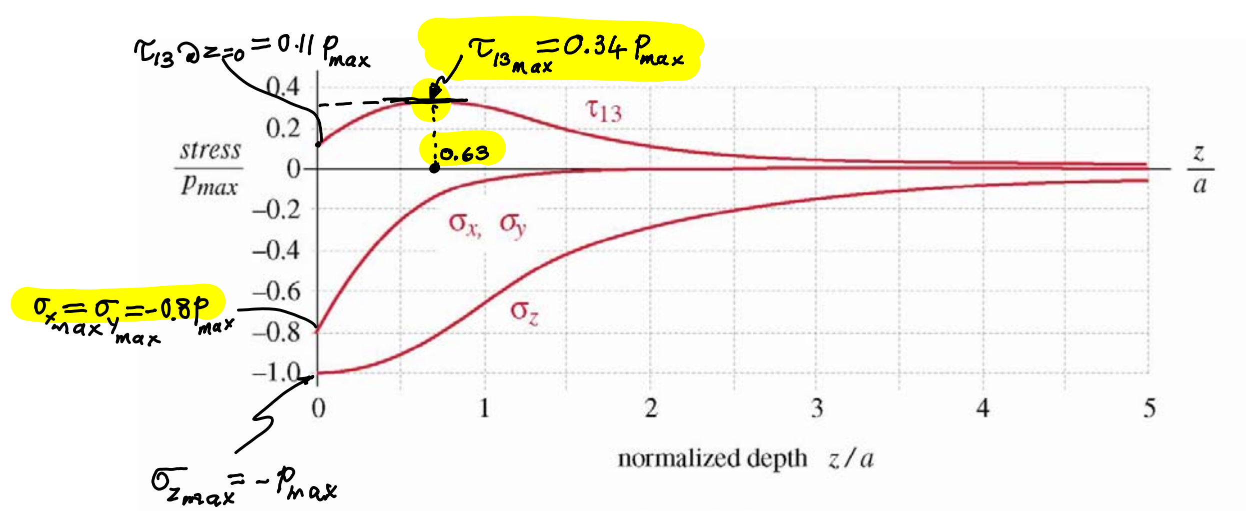


Figure 7-15

Normalized Stress Distribution Along the z Axis in Static Spherical Contact - xyz Stresses are Principal.

Note that all stresses diminish to less than 10% at about z=5a. The fact that τ_{13max} occurs slightly below the surface is believed to be a significant factor in surface fatigue failure. Cracks may initiate below the surface and grow to the point that material above the crack breaks loose to form a pit. Other evidence show that cracks may begin at the surface.

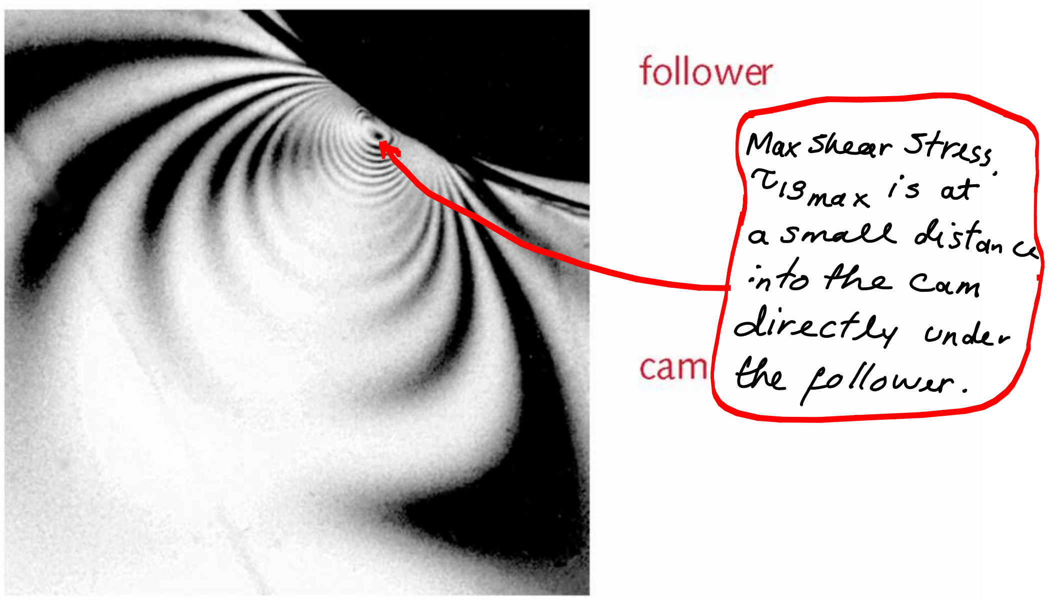


Figure 7-16

Photoelastic Analysis of Contact Stresses Under a Cam Follower Source: V. S. Mahkijani, *Study of Contact Stresses as Developed on a Radial Cam Using Photoelastic Model and Finite Element Analysis.* M.S. Thesis, Worcester Polytechnic Institute, 1984.

It is important to point out that when we move off the centerline of the contact area on the surface of the sphere, the stresses diminish. At the edge of contact area, σ_z is zero but there is a pure shear stress state with magnitude:

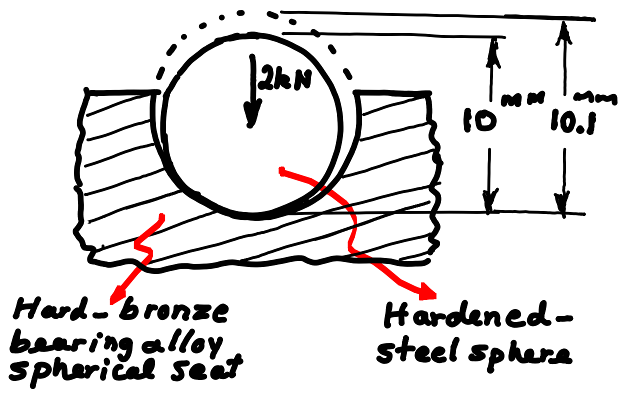
$$\tau_{xy} = \frac{1-2\nu}{3} p_{max} \qquad (7.13a)$$

The two non-zero principal stresses corresponding to this pure shear stress state are

$$\sigma_{1\,edge} = \sigma_{2\,edge} = \frac{1-2\nu}{3} p_{max} \qquad (7.13b)$$

Example- Contact stresses for Ball and Socket Joints

A ball and socket joint has a hardened-steel spherical surface 10 mm in diameter fitting in a hard-bronze bearing alloy spherical seat 10.1 mm in diameter. What maximum contact stress will result from a load of 2000 N?



Let sphere 1 be the steel ball:

Let sphere 2 be the bronze socket:

From (7.9a): $$m_1 + m_2 = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$$

From (7.9b): $$B = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) =$$

From (7.9d): $$a = \sqrt[3]{0.375\,\frac{m_1 + m_2}{B}\,F}$$

From (7.8b): $$p_{max} = \frac{3}{2}\,\frac{F}{\pi a^2}$$

7.9 Cylindrical Contacts

These occur, e.g., in rollers and roller bearings. The cylinders could be both convex, one convex and one concave, or in the limit, a cylinder-on-plane. These could be subject to pure rolling, or rolling and sliding.

In the case of pure rolling, the pressure distribution will be a semi-elliptical prism of half-widths "a" (Fig. 7-11b). If L is the length of contact along the cylinder axis, we have

$$p_{max} = \frac{2F}{\pi a L}, \quad p_{avg} = \frac{F}{area} = \frac{F}{2aL}, \quad p_{max} = \frac{4}{\pi} p_{avg} = 1.273\, p_{avg} \qquad (7.14)$$

and

$$a = \sqrt{\frac{2}{\pi} \frac{m_1 + m_2}{B} \frac{F}{L}} \qquad (7.15)$$

The pressure distribution is elliptical: $p = p_{max}\sqrt{1 - \frac{x^2}{a^2}}$ (7.16)

Max. normal stresses occur at $z = 0$:

$$\sigma_{x_{max}} = \sigma_{z_{max}} = -p_{max}, \quad \sigma_{y_{max}} = -2\nu p_{max} \qquad (7.17a)$$

Again the max. shear stress τ_{13} occurs beneath the surface at

$$z_{@z max} = 0.786a, \quad \text{and} \quad \tau_{13_{max}} = 0.304\, p_{max} \qquad (7.17b)$$

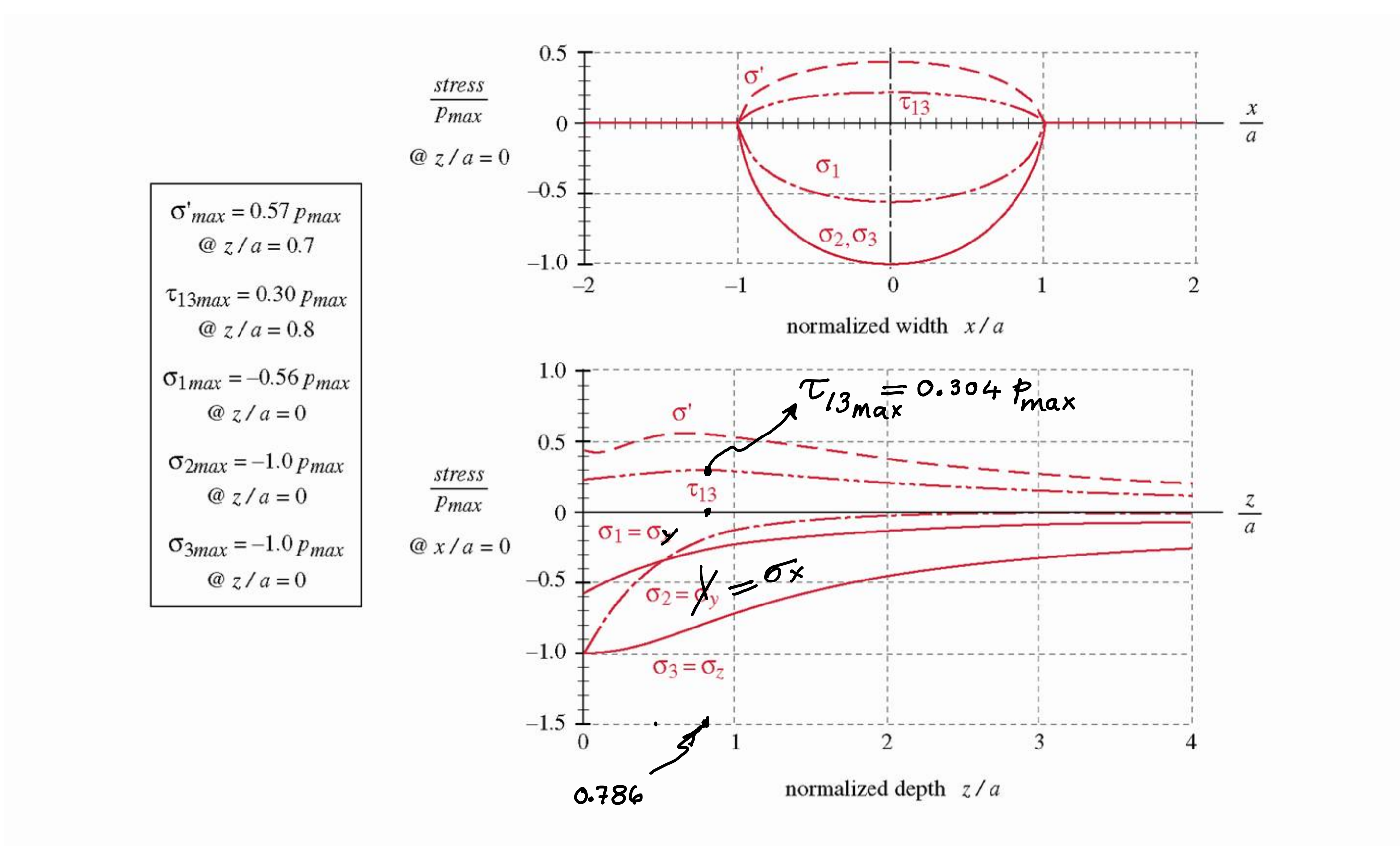


Figure 7-17

Principal, Maximum Shear, and von Mises Stress Distributions for Steel Cylinders in Static Loading or Pure Rolling.

Reading Assignment: Sections 7-10 to 7-14.

Part II - Machine Design

Chapter 10 - Shafts, Keys, and Couplings

A "shaft" is a rotating member, usually of circular cross-section, used to transmit power or motion. Gears, sprockets, pulleys, and cams are usually attached to shafts by means of pins, keys, snap rings, etc. An "axle" is a non-rotating member that carries no torque and is used to support rotating wheels, pulleys, and the like. The "automotive axle", however, is not really a true axle; the term is a carry-over from the horse-and-buggy era, when the wheel rotated on non-rotating members.

10.3 Shaft Materials

Minimizing deflection is an important criterion in shaft design. Hence, steel is a logical choice for a shaft material because of its high modulus of elasticity. Cast iron is sometimes also used, especially if gears or other attachments are integrally cast with the shaft. In marine or other corrosive environments bronze or stainless steel is used. A good practice is to start with an inexpensive, low or medium carbon steel for the first time through the design calculations. If strength considerations turn out to dominate over deflection, then a higher strength material should be tried, allowing the shaft sizes to be reduced until excess deflection becomes an issue. The cost of the material and its processing must be weighed against the need for smaller shaft diameter.

10.4 Shaft Power

The power transmitted through a shaft is the product of torque, T, and angular velocity, ω:

$$P = T\omega \tag{10.1a}$$

where ω is in radian per unit time. When both torque and ω are varying with time, the average power is given by:

$$P_{avg} = T_{avg}\,\omega_{avg} \tag{10.1b}$$

The unit of power is N.m/s = J/s = W, however, it is often given in terms of "horse-power", hp, where

1 hp = 550 lb-ft/s = 6600 lb-in/s = 745.7 W

Note that for a given power, the slower the speed, the higher the torque.

10.2 Attachments and Stress Concentrations

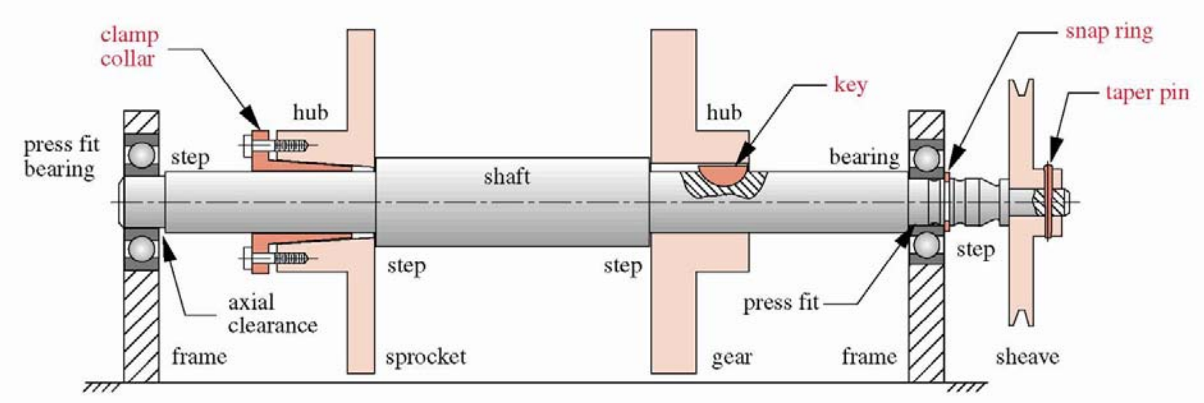


Figure 9-2
Various Methods to Attach Elements to Shafts.

See text for detailed description of various ways to attach elements to shafts.

10.1 & 10.5 Shaft Loads & 10.6 Shaft Stresses

The most general loading for a shaft is the combination of a fluctuating torque and a fluctuating moment. As discussed in Chapter 6, this produces a multiaxial stress state. We must find these applied stresses at all critical points of interest along the shaft. Denoting the mean and alternating components of the bending moment by M_m and M_a, the largest alternating and mean bending stresses are found from

$$\sigma_a = k_f \frac{M_a c}{I} \quad \text{and} \quad \sigma_m = k_{fm} \frac{M_m c}{I} . \qquad (10.2a)$$

Note the change in notation from K_f and K_{fm} in earlier chapters to k_f and k_{fm} here. These stresses occur at the outside surface of the shaft. Noting that $c = r = d/2$ and $I = \pi d^4/64$),

$$\sigma_a = k_f \frac{32 M_a}{\pi d^3} \quad \text{and} \quad \sigma_m = k_{fm} \frac{32 M_m}{\pi d^3} \qquad (10.2c)$$

where d is the local shaft diameter at the cross section of interest. Similarly, the alternating and mean torsional shear stresses are given by

$$\tau_a = k_{fs} \frac{T_a r}{J} \quad \text{and} \quad \tau_m = k_{fsm} \frac{T_m r}{J} \qquad (10.3a)$$

For a solid round shaft, $r = d/2$ and $J = \pi d^4/32$, so that

$$\tau_a = k_{fs} \frac{16 T_a}{\pi d^3} \quad \text{and} \quad \tau_m = k_{fsm} \frac{16 T_m}{\pi d^3} \qquad (10.3c)$$

A tensile axial load F_z, if any is present, will typically have only a mean component (such as weight) and is found from

$$\sigma_{m_{axial}} = k_{fm} \frac{F_z}{A} = k_{fm} \frac{F_z}{\pi d^2} \qquad (10.4)$$